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OPTIMUM REPLACMENT POLICY DECISIONS IN CERTAIN REPLACEMENT MODELS UNDER VARYING CONDITIONS

ABSTRACT

Replacement problem and associated models in O.R. applications can play a very crucial role for taking management decisions. Optimal replacement policy is to be considered for taking a decision for the relevant cases for operations management. There are many studies carried out in this direction. Here an approach for considering variable maintenance cost and also varying resale value is discussed with applications and further studies that may be possible are indicated.

Some Factors effective for Maintenance Cost:

(1) Robustness of item , equipment, machine

(2) Period of usage

(3) Quality material of the item

(4) Type of machine used for the item

(5) Utility of the item

(6) Frequency of usage of the item

(7) Maintenance cost during earlier periods

(8) Machine Productivity for the usage of the item

(9) Accidental situations affecting normal usage

(10) User's attitude or approach for the item etc.

Some Factors effective for Resale Value :

(1) Market condition of the item
(2) Substitution effect due to new technological innovations
(3) Demand for the item
(4) Period of usage
(5) Scarcity situation for the item
(6) Brand or type of product for the item used

(7) Name and reputation of the company producing the item
(8) Condition of the item
(9) Reliability of the item
(10) Repair or Renewal situation for the item etc.

First Model

When the value of money does not change with the passage of time .

A(t)= Maintenance cost function (It is a real ,single valued, monotonically non-decreasing function)
S(t)=Resale value function (It is a real ,single valued, monotonically non-increasing function)
C = Capital cost
ΣA(t) is cumulative M.C. function
ATCn = Average total cost for period n
ATCn = [C-S(t)+ ΣA(t)]/n
Find n* so that ATCn is minimum.

Continuous case : $\frac{dATCn}{dn} = 0$ (N.C.)

$$\left[\frac{dATC_n}{d_n}\right]_{n^*} > 0 \qquad \text{(S.C.)}$$

Discrete case : Find n* such that $ATC_n \le ATC_{n+1}$ and $ATC_n \le ATC_{n-1}$.

COST STRUCTURE AND ANALISIS

CASE	M.C/C.M.C./RSV	ATC_n	SOLUTION/REMARKS
1.1(C)	$A(t)= \alpha_0+ \alpha_1 t$, $0\le t\le n$, $\alpha_0,\alpha_1> 0$ CMC = $\alpha_0 n+ \alpha_1 n^2/2$ RSV =S	$\frac{c-s}{n}+\alpha_0+\frac{\alpha_1 n}{2}$	$n^* = \sqrt{\frac{2(c-s)}{\alpha_1}}$ $[S.C.] \quad \frac{2(c-s)}{n^3} > 0$
1.2(D)	$A(t)= \alpha_0+ \alpha_1 t$, t=1,2,3…n , $\alpha_0,\alpha_1> 0$ CMC = $\alpha_0 n+ \alpha_1 n(n+1)/2$ RSV =S	$\frac{c-s}{n}+\alpha_0+\frac{\alpha_1(n+1)}{2}$	Inequality to find n^*, $n(n-1) \le \frac{2(c-s)}{\alpha_1} \le n(n+1)$
2.1(C)	$A(t)= \alpha_0+ \alpha_1 t + \alpha_2 t^2$, $0\le t\le n$ CMC = $\alpha_0 n+ \alpha_1 n^2/2 + \alpha_2 n^3/3$ RSV =S	$\frac{c-s}{n}+\alpha_0+\frac{\alpha_1 n}{2}+\frac{\alpha_2 n^2}{3}$	cubic equation to find n^*, $[S.C.]$ $\frac{2(c-s)}{n^3}+\frac{2\alpha_2}{3} > 0$

2.2(D)	A(t)= $\alpha_0+\alpha_1 t+\alpha_2 t^2$, t=1,2,3…n CMC = $\alpha_0 n+\alpha_1 n(n+1)/2+\alpha_2 n(n+1)(2n+1)/6$ RSV =S	$\frac{c-s}{n}+\alpha_0+\alpha_1+\frac{\alpha_1(n+1)}{2}$ $+\frac{\alpha_2(2n^2+3n+1)}{6}$	Inequality to find n^*, $\frac{\alpha_1}{2}+\frac{\alpha_2}{2}(4n+5)\geq\frac{c-s}{n(n+1)}$ $\frac{\alpha_1}{2}+\frac{\alpha_2}{6}(4n+1)\leq\frac{c-s}{n(n-1)}$
3.1(C)	A(t)= $a_0+a_1 t+a_2\log t$, $1\leq t\leq n$; $a_0,a_1,a_2>0$ CMC = $a_0 n+a_1 n^2/2+a_2 n(\log n-1)-(a_0+a_1/2-a_2)$ RSV =S(t)= $\alpha-\beta t$, $1\leq t\leq n$, $\alpha>0,\beta>0$	$\frac{C-(\alpha-\beta n)}{n}+\frac{CMC}{n}$	N.C. gives, $An^2+Bn+K=0$ $n^*=\frac{-a_2+\sqrt{a_2^{\ 2}-2a_1K}}{a_1}$ $A=\frac{a_1}{2}$; $B=a_2$ $K=\left[\alpha+a_0+\frac{a_1}{2}-(a_2+c)\right]$ [S.C.] $\frac{c-(\alpha+a_0+\frac{a_1}{2}-a_2)}{n}>\frac{a_2}{2}$
3.2(D)	A(t)= $a_0+a_1 t+a_2\log t$, t=1,2,3…n, $a_0,a_1,a_2>0$ CMC = $a_0 n+a_1 n(n+1)/2+a_2\,\Sigma\log t$ RSV =S(t)= $\alpha-\beta t$, t=1,2,3,…n , $\alpha>0,\beta>0$	$\frac{c-\alpha}{n}+(a_0+\beta)+\frac{a_1(n+1)}{2}+a_2L_n$ $L_n=\sum_1^n\log t$	Inequality to determine n^* $(n-1)\left[\frac{a_1}{2}+a_2\Delta L_{n-1}\right]$ $\leq\frac{c-\alpha}{n}\leq(n+1)\left[\frac{a_1}{2}+a_2\Delta L_n\right]$ $\Delta L_n=L_{n+1}-L_n$
CASE	M.C/C.M.C./RSV	ATC_n	SOLUTION/REMARKS
4.1(C)	A(t)= $a_0+a_1 t+a_2 1/t$, $1\leq t\leq n$, $a_0,a_1,a_2>0$ CMC = $a_0 n+a_1 n^2/2+a_2\log n-(a_0+a_1/2)$ RSV =S(t)= $\alpha-\beta t$, $1\leq t\leq n$, $\alpha>0,\beta>0$	$ATC_n=\frac{C-(\alpha-\beta n)}{n}+\frac{CMC}{n}$	N.C. gives f(n)= $\frac{a_1n^2}{2}-a_2\log n$ $P=\alpha-a_{0-}\frac{a_1}{2}+a_2$ Approx logn □ $n-\frac{n^2}{2}$ gives $An_0^2+Bn_0+K=0$; $A=(a_1+a_2)/2$;$B=-a_2$;$K=P-C$ $n^*=\frac{a_2+\sqrt{a_2^{\ 2}+2(a_1+a_2)K}}{(a_1+a_2)}$ [S.C.] $\log n^*>(\frac{1}{a_2})(\frac{3a_2}{2}+\alpha+a_0+\frac{a_1}{2}-c)$ Note If $a_1=0, A(t)=a_0+\frac{a_2}{t}$ $n^*=\frac{a_2+\sqrt{a_2^{\ 2}+2a_2K}}{a_2}$ If $\delta=\frac{\alpha}{\beta}$,then

			As $\delta \to 0$, $n^* \to e^{\left(1-\frac{c-s}{\beta}\right)}$ As $\delta \to 1, n^* \to e^{\left(-\frac{(c-s)}{\beta}\right)}$
4.2(D)	A(t)= $a_0+ a_1t + a_2/ t$, t=1,2,3…n, $a_0,a_1, a_2>0$ CMC = $a_0n+ a_1n(n+1)/2 + a_2 \Sigma 1/t$ RSV =S(t)= α-β t , t=1,2,3,…n , α>0,β>0	$ATC_n = \frac{C-\alpha}{n} + \frac{(a_0+\beta)}{n} + \frac{a_1(n+1)}{2} + a_2H_n$ where $H_n = H.M. = \frac{1}{n}\sum_1^n \frac{1}{t}$	ATC_n $\leq A\,\delta \to 1, n^* \to e^{-(\frac{c-s}{\beta})}$ TCn-1 ATCn$\leq$ATCn+1 $gives(n-1)[\frac{a_1}{2} + a_2\Delta H_{n-1}]$ $\leq \left(\frac{c-\alpha}{n}\right) \leq [\frac{a_1}{2} + a_2\Delta H_n]$ Where $\Delta H_n = H_{n+1} - H_n]$ $\Delta H_n = H_{n+1} - H_n]$
CASE	M.C/C.M.C./RSV	ATC_n	SOLUTION/REMARKS
5.2(D)	A(t)= $a_0+ a_1t + a_2e^{\lambda t}$, t=1,2,3…n, $a_0,a_1, a_2, \lambda >0$ CMC = $a_0n+ a_1n(n+1)/2 + a_2 \Sigma e^{\lambda t}$ RSV =S(t)= α-β t , t=1,2,3,…n , α>0,β>0	$ATC_n = \frac{C-(\alpha-\beta_n)}{n} + \frac{CMC}{n}$	n* is determine from the inequality $(n-1)[\frac{a_1}{2} + a_2\Delta Q_{n-1}] \leq \frac{c-\alpha}{n}$ $\leq (n+1)[\frac{a_1}{2} + a_2\Delta Q_{n-1}]$ where $Q_n = \frac{1}{n}\sum_1^n e^{\lambda t}$ $\Delta Q_n = Q_{n+1} - Q_n$ $\Delta Q_{n-1} = Q_n - Q_{n-1}$
6 (C)	Two diff. equations as given by $\frac{dA(t)}{dt} = \frac{\alpha}{S(t)} + \beta$ α>0,β>0 A(0)=A_0 and S(0)=S_0 $\frac{dS(t)}{dt} = -\lambda S(t)$, λ >0 which gives S(t)=$S_0e^{-\lambda t}$ and $A(t) = A_0 + \beta t + \frac{\alpha}{\lambda S_0}(e^{\lambda t} - 1)$ $CMC = \int_0^n A(t)dt$	$ATC_n = (A_0 - \frac{\alpha}{\lambda S_0}) + \frac{\beta n}{2}$ $- \frac{\alpha}{n\lambda^2 S_0} + \frac{\alpha}{\lambda^2 S_0}(\frac{e^{\lambda n}}{n})$	N.C. gives f(n/α,β,λ,S_0)=0 particular case (I) if β=0 as $n^* \to \infty$, $ATC_n^* \to R_0 - \frac{\alpha}{\lambda S_0}$ (ii) If S(t)=S_0 then A(t)=$(\frac{\alpha}{S_0} + \beta)t + A_0$ CMC= $R_0n + (\frac{\alpha}{S_0} + \beta)\frac{n^2}{2}$ Hence , $n^* = \sqrt{\frac{c-s_0}{\left(\frac{\alpha}{s_0} + \beta\right)}}$

	$CMC = A_0 n + \frac{\beta n^2}{2} + \frac{\alpha}{\lambda^2 S_0} e^{\lambda n} - \frac{\alpha\, n}{\lambda\, S_0} - \frac{\alpha}{\lambda^2 S_0}$		If β=0, n*= $n^* = \sqrt{\frac{s_0(c-s_0)}{\alpha}}$
7 (C)	Two diff. equations as given by $\frac{dA(t)}{dt} = \frac{\alpha t}{S(t)} + \beta$ α>0,β>0 A(0)=A_0 and S(0)=S_0 $\frac{dS(t)}{dt} = -\lambda S(t)$, λ >0 which gives S(t)=$S_0 e^{-\lambda t}$ and $A(t) = (A_0 + \frac{\alpha}{S_0\alpha^2}) + \beta t + \frac{\alpha e^{\lambda t}}{\lambda^2 S_0}(\lambda t - 1)$ $CMC = \int_0^n A(t)dt$	$ATC_n = \frac{(C - S_0 e^{-\lambda n}) + CMC}{n}$	N.C. gives f(n/α,β,λ,S_0)=0 particular case $e^{\lambda t} \approx 1 + \lambda t$ gives ,$A(t)=A_1+A_2+A_3t^2$ Where, $A_1=A_0+ a/S_0 \lambda^2- \alpha/ S\lambda^2$ $A_2=\beta$ and $A_3=\alpha/S_0$ Hence the solution
CASE	M.C/C.M.C./RSV	ATC_n	SOLUTION/REMARKS
8 (D)	A(t)=a_0A(t-1) , $a_0>1$ A(0)=a ,a>0, t=1,2,3…n $CMC = \frac{a}{(a_0-1)}(a_0^n + a_0 - 2)$ RSV =S(t)=b_0S(t-1) , 0< b_0<1 , $S_0=b_0K$,K>0	$ATC_n = \frac{(C - Kb_0^n)}{n} + \frac{\left(\frac{a}{a_0-1}\right)(a_0^n + a_0 - 2)}{n}$	Inequality $ATC_n \leq ATC_{n-1}$ and $ATC_n \leq ATC_{n+1}$ determines n*
9 (D)	A(t)=a_0A(t-1)+a_1t , a_0>0 A(0)=a ,a>0, t=1,2,3…n Which gives ,A(t)=Ea_0^t +Bt+D, a>0 Where $E = [a + \frac{a_0 a_1}{(a_0-1)^2}]$ $B = -[\frac{a_1}{(a_0-1)}]$, $D = -[\frac{a_0 a_1}{(a_0-1)^2}]$ RSV =S(t)=p_0S(t-1) , 0<p_0<1 , $S_0=p_0^t K$,K>0	$ATC_n = \frac{C - S(n)}{n} + \frac{1}{n}\sum_1^n A(t)$ $ATC_n = \frac{C - p_0^n K}{n} + \frac{1}{n}\left[\frac{Ea_0(a_0^n-1)}{(a_0-1)} + \frac{Bn(n+1)}{2} + nD\right]$	Inequality $ATC_n \leq ATC_{n-1}$ and $ATC_n \leq ATC_{n+1}$ determines n*

Applications:

Some models developed here are utilized for certain practical fields like industrial engineering, automobile industry, machine shop processes etc.

These applications are used for certain products as mentioned below

(1) Lathe cutter

(2) Cotter pin

(3) Engine valve

(4) Machine Bearing

(5) Spark plug

(6) Conveyer Belt pulley

(7) Stentner

(8) Cylindrical drum used in machine

(9) Air filter

(10) Crankshaft etc.

Further studies :

(1) Type of models discussed may be extended for the case $A(t)=f(n,x_1,x_2,x_3,\ldots x_k)$

Where x_1=time,x_2=frequency of usage,x_3=robustness, x_4=representing quality of item , etc.

A multiple regression model may be framed. Stepwise regression can help for solutions. Model could be non-linear in nature.

(2) When the value of money changes as per passage of time, the above models may be considered.

Published work :

Research papers based upon above discussions are published in the journals like- Research Bulletin, Journal of Industrial Engineering , Sahyog, Sankhya vignan, Gujarat statistical Review, Vishleshan, Arthsankalan, Journal of I.C.W.A., Decision etc.

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